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3.6 LITERAL EQUATIONS

Name: ______________________________

CHANGING THE SUBJECT
or ISOLATING VARIABLES
IN FORMULAS

Warm Up:

Solve each equation for x:

Ex 1:

Ex 2:

Are there any differences in how we solved these?

What is an Equation? An equation says that two things are equal. It will have an equals sign "=" like this:

$$x \quad + \quad 2 \quad = \quad 6$$

That equations says:
what is on the left (x + 2) is equal to what is on the right (6)

What is a Formula?

- A formula will have **more than one variable**.

These are all equations, but only some are formulas:

$\mathbf{x} = 2\mathbf{y} - 7$	Formula (relating **x** and **y**)
$\mathbf{a}^2 + \mathbf{b}^2 = \mathbf{c}^2$	Formula (relating **a**, **b** and **c**)
$\mathbf{x}/2 + 7 = 0$	Not a Formula (just an equation)

- A formula is a special type of equation that shows the relationship between different **variables**.

Figure	***Area Formula***	***Perimeter Formula***	***Variables***
Square	$A = s^2$	$P = 4s$	s = length of side
Rectangle	$A = lw$	$P = 2(l + w)$	l = length w = width
Triangle	$A = \frac{1}{2}bh$	$P = a + b + c$	a, c = sides b = base h = height
Circle	$A = \pi r^2$	$P = 2\pi r$	r = radius

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Subject of a Formula

The "subject" of a formula is the single variable (usually on the left of the "=") that everything else is equal to.

Example: in the formula

$$s = ut + \tfrac{1}{2} at^2$$

"s" is the subject of the formula

Changing the Subject

One of the very powerful things that Algebra can do is to "rearrange" a formula so that another variable is the subject. Changing the subject of a formula can be a huge time saver.

Rearrange the volume of a box formula (**V = lwh**) so that the width is the subject:

Start with:	**V = lwh**
divide both sides by h:	**V/h = lw**
divide both sides by l:	**V/(hl) = w**
swap sides:	**w = V/(hl)**

So now when you want a box with a volume of $12m^3$, a length of 2m, and a height of 2m, you can calculate its width:

$$w = V/(hl)$$

$$w = 12m^3 / (2m \times 2m) = 12/4 = 3m$$

Why Should Change the Subject of a Formula?

Believe it or not, there are many good reasons to develop your ability to rearrange formulas that are important to the geosciences. It can save time, help you with units and save some brain space! Here are some reasons to develop your equation manipulation skills (in no particular order):

- Equations are easier to handle *before* inserting numbers! And, if you can isolate a variable on one side of the equation, it is applicable to every similar problem that asks you to solve for that variable!
- If you know how to manipulate equations, you only have to remember one equation that has all the variables of question in it - you can manipulate it to solve for any other variable! This means less memorization!
- Manipulating equations can help you keep track of (or figure out) units on a number. Because units are defined by the equations, if you manipulate, plug in numbers and cancel units, you'll end up with exactly the right units (for a given variable)!

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Example 1

Solve $P = 2l + 2w$ for w.

Do	Undo

Example 2

The area A of a sector (a pie-wedge-shaped section) of a circle is given by:

$$A = \frac{\pi r^2 S}{360}$$

...where r is the radius of the circle and S is the angle measure (in degrees) of the sector. Solve this equation for S.

Do	Undo

Example 3 - Special Case

When you can't isolate the desired variable because it is a factor in two or more terms, collect those terms together on one side of the "equals" sign, factor out the desired variable, and then divide off whatever is left.

Solve $Q = 3a + 5ac$ for a

Ex 4: Area of a Triangle, solve for b

Ex 5: Perimeter of a Rectangle.

Use the following formula for interest and solve the given variables.

A =

Ex 6: Solve for P.

Ex 7: Solve for t.

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Sometimes the variable you need to get by itself is in more than one location.
HOT TIP:

Ex 8: Solve for x.

Ex 9: Solve for x.

You Try:

1) Solve for c.

2) Solve for x.

SUMMARY:

Now, summarize your notes here!

3.6 PRACTICE PROBLEMS

1) $P = IRT$ (T)

2) $A = 2(L + W)$ (W)

3) $y = 5x - 6$ (x)

4) $2x - 3y = 8$ (y)

5) $\frac{x+y}{3} = 5$ (x)

6) $y = mx + b$ (b)

7) $ax + by = c$ (y)

8) $A = h(b + c)$ (b)

9) $V = LWH$ (L)

10) $A = 4r^2$ (r^2)

11) $V = \pi r^2 h$ (h)

12) $7x - y = 14$ (x)

13) $A = \frac{x+y}{2}$ (y)

14) $R = \frac{E}{I}$ (I)

15) $x = \frac{yz}{6}$ (z)

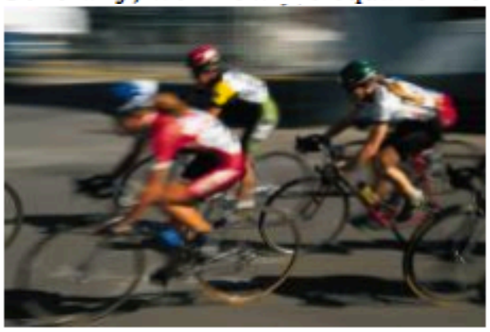
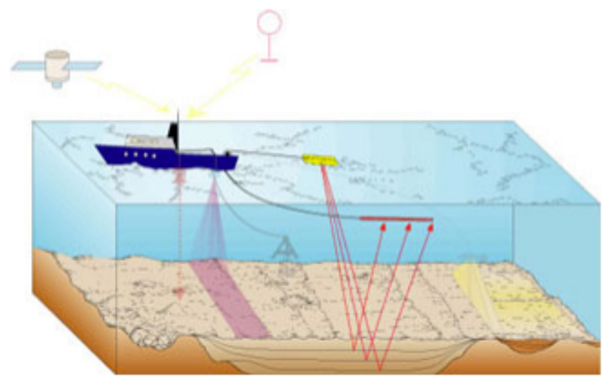
16) $A = \frac{r}{2L}$ (L)

17) $A = \frac{a+b+c}{3}$ (b)

18) $12x - 4y = 20$ (y)

19) $x = \frac{2y-z}{4}$ (z)

20) $P = \frac{R-C}{N}$ (R)

Velocity, distance, and time	
21) Generally, we know the equation for velocity (a rate) to be:  $v = \frac{d}{t}$ Where v = velocity, d = distance and t = time. This equation can be rearranged so that you have an equation for distance (d) and time (t).	
a)	**Rearrange the velocity equation to create an equation for distance (d).** **Rearrange the velocity equation to create an equation for time (t).**
Use the equations that you manipulated above to solve the following problems:	
b)	A wave traveling downward from the surface of the ocean at 1.5 km/sec takes six seconds to reflect off the ocean floor. How deep is the ocean at that site? 
C)	Imagine that you are working with Ms. Homeowner to understand the ground water flow in her area. She is particularly interested in an underground tank that is located 2.6 km from her home. You have measured the velocity of the groundwater to be 0.033km/day. About how long will it take any contaminants leaking from the tank to reach Ms. Homeowner's well?

Density	
22) Density plays an important role in our understanding of the physical properties of Earth materials. The equation for density is similar to that for velocity and, as such, it can be manipulated so that you can solve for any of the variables involved. The next few problems utilize the equation for density: which can be shortened to: $$\rho = \frac{m}{V}$$ **Units for mass are grams** **Units for density are grams per cubic centimeter** **Units for volume are cubic centimeters**	
a)	: Manipulate (rearrange) the density equation (above) to create an equation for mass. :Manipulate the density equation to create an equation for volume.
Use the equations that you manipulated above to solve the following problems:	
b)	The outer core has a density of about 2.75 g/cm^3. What volume would 1000g of the outter core occupy?
e)	The outer core has a density of about 10.5 g/cm^3. What volume would 1000g of the outter core occupy?

Solve the formula below for the indicated variable.		
23) $S = 5r^2t$, for t	Do Undo	$S = 5r^2t$
24) $T = \frac{2U}{E}$, for U	Do Undo	$T = \frac{2U}{E}$
25) $A = \frac{1}{2}bh$, for h	Do Undo	$A = \frac{1}{2}bh$
26) $P = 2l + 2w$, for l	Do Undo	$P = 2l + 2w$
27) $(y - y_1) = m(x - x_1)$, for m	Do Undo	$(y - y_1) = m(x - x_1)$
30) $C = 2\pi r$, for r.	Do Undo	$C = 2\pi r$

31) $Ax + By = C$, for y	Do Undo	$Ax + By = C$
32) $C = \frac{5}{9}(F - 32)$, for F	Do Undo	$C = \frac{5}{9}(F - 32)$
33) $A = \frac{1}{2}(b_1 + b_2)$, for b_1	Do Undo	$A = \frac{1}{2}(b_1 + b_2)$
34) $\frac{3x + y}{c} = 4$, for x.	Do Undo	$\frac{3x + y}{c} = 4$
35) $(y - y_1) = m(x - x_1)$, for m	Do Undo	$(y - y_1) = m(x - x_1)$

36) $C = 2\pi r$, for r.	Do Undo	$C = 2\pi r$
37) $Ax + By = C$, for y	Do Undo	$Ax + By = C$
38) $L = a + (n - 1)d$ for n	Do Undo	
39) $D = \frac{R(100 - x)}{100}$ for R	Do Undo	
40) $D = \frac{C - S}{n}$ for S	Do Undo	

SKILLS REVIEW		
1. Simplify. $\frac{4}{9}\cdot\frac{27}{32}$	2. Simplify. $3h-14-h+10$	3. Simplify. $3-4(x+7)$
4. Simplify. $\frac{2}{3}\left(6x-\frac{1}{2}\right)$	5. Simplify. $8\sqrt{3+6}$	6. Simplify. $-\lvert 3+(-4)\rvert$

3.6 **APPLICATIONS**

1. Solve for T.

$$V=\frac{KT}{P} \quad \text{for } T$$

2. The volume enclosed by a cone is given by the formula

$$V=\frac{1}{3}\pi r^2 h$$

Where r is the radius of the circular base of the cone and h is its height. In the figure above, drag the orange dots to change the radius and height of the cone and note how the formula is used to calculate the volume.

a. Make the radius, r, the new subject of the equation by isolating the variable.

b. Find the radius given that the volume of the cone is 9377.1 cubic centimeters, he height of the cone is 31 centimeters, and π = 3.14.

3. Make up your own formula. Then, isolate a one of the variables (making it the new subject of the formula). Make sure to show you work.

Solve the equation or formula for the indicated variable.

4. The formula for the time a traffic light remains yellow is $t = \frac{1}{8}s + 1$, where t is the time in seconds and s is the speed limit in miles per hour.

   a. Solve the equation for s.

   $t = \frac{1}{8}s + 1$

Do	Undo

   $s =$

   b. What is the speed limit at a traffic light that remains yellow for 4.5 seconds?

5. The volume V of a cylinder is given by the formula $V = \pi r^2 h$ where r is the radius of the cylinder and h is the height of the cylinder.

   a. Solve the formula for h.

   $V = \pi r^2 h$

Do	Undo

   $h =$

   b. What is the height given that $V = 87.92 m^3$
   $\pi = 3.14$
   $r = 2m$

6. Mr. Kelly has attended many national and local teaching workshops and conferences and has heard all about how students should discover mathematics for themselves. He became so interested in this teaching methodology that he went home and "discovered" his own theorem for finding the sides of a right triangle. He found the following formula when you have a right triangle with shortest side A, medium side B and longest side C: $A=\sqrt{C^2-B^2}$

   a) Solve the Kellian Theorem (sounds good right?) for B.

   b) Mr. Brust points out that Mr. Kelly just stole the Pythagorean Theorem, which had already been discovered a few years earlier. Solve the Pythagorean Theorem ($A^2+B^2=C^2$) for a to see if Mr. Brust is right.

Extension:

1. A) Solve for E $W+ES-L=EY$	2. B) Find the value of E, Given: $E=x,\ W=44,\ L=1,\ S=7,\ \&\ Y=3$

2. Solve for a. $x=\dfrac{a+b+c}{ab}$	3. Solve for b $y=x\left(\dfrac{ab}{a-b}\right)$	4. Solve for T $M+AT=T$