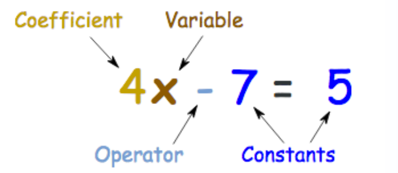
4.4 Solve Equations with rational Coefficients

Name: ____________________

Write your questions here!

Key Vocabulary

Coefficient:

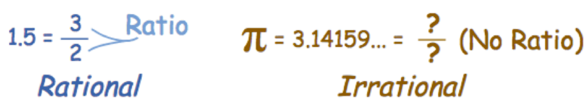


Variables with no number have a coefficient of 1.

Example: x is really 1x.

Rational Numbers:

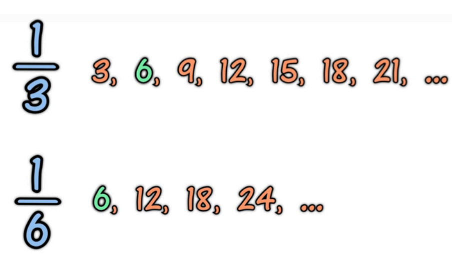
Examples:
- 1/2 is a rational number (1 divided by 2, or the ratio of 1 to 2)
- 0.75 is a rational number (3/4)
- 1 is a rational number (1/1)
- 2 is a rational number (2/1)
- 2.12 is a rational number (212/100)
- −6.6 is a rational number (−66/10)



Rational Coefficients:

Examples:
- $7 = \frac{1}{2}x + \frac{3}{4}x - \frac{2}{3}x$
- $0.73x + 7.4 = 19.5 - 0.1x$

Least Common Denominator (or LCD):



Write your questions here!

Lets try solving some equations with rational coefficients.

Examples

Regular way

$$\frac{2}{3}n - 6 = 22$$

Don't take notes on this, just watch.

Urquhart Method:

$$\frac{2}{3}n - 6 = 22$$

LCD –

Regular way

$$\frac{-1}{2}x + \frac{1}{3} = \frac{1}{5}x - \frac{1}{6}$$

Don't take notes on this, just watch.

Urquhart Method:

$$\frac{-1}{2}x + \frac{1}{3} = \frac{1}{5}x - \frac{1}{6}$$

LCD –

Urquhart Method:

$$\frac{2}{5}x + 2 = \frac{3}{7}$$

LCD –

$7 = \frac{1}{2}x + \frac{3}{4}x - \frac{2}{3}x$

$\frac{1}{2}(y - 5) = \frac{1}{4}(y - 1)$

$0.8x - 5 = 7$

$0.06x + 0.02 = 0.25x - 1.5$

$0.25x + 0.05(x + 3) = 2.85$

Now, summarize your notes here!

SUMMARY:

SOLVE EQUATIONS WITH FRACTIONS

Step 1 – Find the LCD
Step 2 – Multiply both sides of the equation by the LCD
Step 3 – Simplify the coefficients
Step 4 – Solve the equation
Step 5 – Check you answer.

Solve each equation by first clearing the fractions. Verify the solution.

1. $\frac{x}{3} = x + 4$

2. $x + \frac{3x}{4} = 7$

3. $m - 3 = \frac{4}{5}m - 2$

4. $2x - 1 = \frac{3}{4}x + 9$

5. $z + \frac{3}{5} = \frac{z}{5}$

6. $\frac{3w}{2} + 1 = w + \frac{9}{2}$

7. $\frac{4}{9}w + 5 = \frac{5}{9}$

8. $6 + \frac{x}{5} = \frac{4x}{5} - 3$

FRACTIONS WITH DIFFERENT DENOMINATORS

If the denominators are different, we must identify the LCD before we multiply. Then, to clear the fractions, we must multiply each side by the LCD.

9. $\frac{5y}{2} - 9 = \frac{2y}{3} + 2$

10. $\frac{3y}{4} - 6 = \frac{y}{8} + 4$

11. $p - \frac{p}{6} = \frac{p}{3} + 2$

12. $\frac{x}{9} + \frac{4}{3} = \frac{1}{2}x - 1$

13. $\frac{3x}{5} + \frac{1}{6} = \frac{x}{2} - \frac{1}{3}$

14. $\frac{y}{4} + \frac{1}{12} = \frac{y}{3} - \frac{1}{6}$

15. $\frac{5t}{9} - \frac{3}{4} = \frac{1}{12} + \frac{5t}{8}$

16. $\frac{3x}{20} + \frac{1}{10} = \frac{x}{4} - \frac{1}{5}$

Equations Containing Decimals

Recall from section 2.3 that terminating decimals are rational numbers (fractions) in which the denominators are powers of 10, such as 10, 100, and so on.

For example, $0.3 = \frac{3}{10}$ and $0.25 = \frac{25}{100}$.

Consider an equation that contains these two fraction: $\frac{3}{10}\ x = \frac{25}{100}\ x + 1$. We can clear the fractions by multiplying each side by the LCD of 100, changing it to an equation of integers: $30x = 25x + 100$.

If this same equation is written with decimals instead of fractions, it would be $0.3x = 0.25x + 1$. Because this is the same equation, we also can multiply each side by 100, but this time we will *clear the decimals.*

One major distinction, when clearing decimals, is to prepare the equation by first *writing each constant and coefficient with the same number of decimal places.*

For example, each number in the equation $0.3x = 0.25x + 1$ can be written with two decimal places:

- For 0.3, we can place one zero at the end of the number: $0.3 = 0.30$
- For 1, we can place a decimal point and two zeros at the end of the number: $1 = 1.00$
- 0.25 already has two decimal places, so no change is necessary.

The equation becomes $0.30x = 0.25x + 1.00$. Now having two decimal places, each number is in terms of *hundredths*, and we can clear the decimals by multiplying each side by 100:

$$\mathbf{100(}0.30x\mathbf{)} = \mathbf{100(}0.25x + 1.00\mathbf{)}$$

Multiplying by 100 has the effect of moving the decimal point two places to the right.

$$30x = 25x + 100$$

It is now an equation of integers, and we can solve it using the techniques learned earlier in this chapter.

Example For each equation,

- What number of decimal places should each constant and coefficient have?
- Prepare the equation by building up each number, as necessary.
- By what number should we multiply each side of the equation to clear the decimals?

a) $0.4x - 1.2 = 0.15x + 0.8$ b) $0.12y - 1 = 0.095y - 0.9$

Procedure: For each equation, the constant or coefficient with the highest number of decimal places indicates the number of decimal places each should have.

a) 0.15 has two decimal places so we should build up each number to have two decimal places

b) 0.008 has three decimal places so we should build up each number to have three decimal places

		Number of decimal places	New equation	Multiply each side by
Answer:	a)	Two	$0.40x - 1.20 = 0.15x + 0.80$	100
	b)	Three	$0.120y - 1.000 = 0.095y - 0.900$	1,000

9. Using the example above as a reference, find:

    I. The number of decimal places,
    II. The new equation, and
    III. The number you would need to multiply on both sides to clear the decimals for each example below.

	Number of decimal places	New equation	Multiply each side by
a) $2w - 0.4 = 1 + 1.8w$			
b) $0.17k - 0.43 = 0.25k + 0.05$			
c) $0.27v - 1.6 = 0.32v - 2$			
d) $0.1x - 0.006 = 0.08x + 0.134$			

Solve the equation by first clearing the decimals. Verify the solution.

10. $3x = 2.5x + 10$

11. $4x - 12 = 1.5x + 8$

12. $0.6x - 3.2 = 0.4 - 0.3x$

13. $-1.6 - 0.9w = 11.6 + 2.4w$

14. $0.128 - 0.035v = -0.072v + 0.235$

15. $-0.32v + 0.18v = 0.25v - 1.95$

16. $0.3x = 0.25x + 1$

17. $0.3x + 4.2 = 0.1x + 4$

18. $0.4x - 1.2 = 0.15x + 0.8$

19. $4.72n - 0.1 = 8 + 0.67n$

20. $-0.51x - 3.2 = 0.8x + 7.28$

21. $0.12y - 1 = 0.095y - 0.9$

22. $0.1m + 0.008 = 0.06m - 0.172$

23. ***Think Again.***

If an equation contains decimals, why is it helpful for all the constants and coefficients to have the same number of decimal places?

24. **SHIPPING** The Lone Star Shipping Company charges $14 plus $2 a pound to ship an overnight package. Discount Shipping Company charges $20 plus $1.50 a pound to ship an overnight package. For what weight is the charge the same for the two companies?

X	Y_1	Y_2
Number of Pounds	Cost for the Lone Star Shipping Company	Cost for the Discount Shipping Company
1		
2		
3		
4		
5		
….		
20		
…		
x		

Equation: ______________________________

25. **MONEY** Deanna and Lisa are playing games at the arcade. Deanna started with \$15, and the machine she is playing costs \$0.75 per game. Lisa started with \$13, and her machine costs \$0.50 per game. After how many games will the two girls have the same amount of money remaining? *(Make table if necessary)*

26. **MONEY** The Wayside Hotel charges its guests \$1 plus \$0.80 per minute for long distance calls. Across the street, the Blue Sky Hotel charges its guests \$2 plus \$0.75 per minute for long distance calls. Find the length of a call for which the two hotels charge the same amount. *(Make table if necessary)*

SKILLZ REVIEW		
I. Solve for x. $\frac{x-3}{15}=\frac{3}{4}$	II. Solve for x. $\frac{2x-18}{4}=\frac{3x+1}{2}$	III. Convert 4 yards into inches using dimensional analysis.
IV. Name the property show below. $-8(3x-4)=-24x+32$	V. Name the property show below. $13-15=-15+13$	VI. Name the property show below. $7\cdot\frac{1}{7}=1$

Solving Linear Equations: The Ultimate Guidelines

The Preparation:

1. Eliminate any parentheses by distributing.
2. Clear any fractions or decimals by multiplying each side by the equation's LCD.
3. Combine like terms on each individual side.

Isolating the Variable:

4. If necessary, reduce the equation to standard form.
5. If necessary, isolate the variable term then finish solving.

Solve each equation and verify the solution.

1. $\frac{1}{2}\left(x+\frac{2}{3}\right) = 3(x-1)$

2. $0.2(3y-5) = 0.15(2y+3) - 0.85$

3. $\frac{1}{2}\left(2t - \frac{3}{4}\right) + \frac{2}{5} = \frac{4}{5}t$

4. $0.3(x + 5) = 5(0.1 + 0.11x)$

5. $\frac{3}{8}(m + 8) - \frac{3}{16} = 2\left(m + \frac{3}{4}\right) + \frac{1}{2}$

6. $3.75 - 2.5(p + 1) = 0.5p + 4.25$

7. $\frac{1}{6}(1 - 6x) = -\frac{1}{3}\left(6x + \frac{1}{2}\right)$

8. $2.3y = 0.15(2y - 3) - 0.6$

9. ***Think Again.***

Consider the equation $2x + 1 = \frac{1}{4}\left(\frac{1}{2}x + 4\right)$. What is the least common denominator on the right side of this equation?

Think Outside the Box:

Solve each.

10. $\frac{x + 3}{8} - \frac{x}{2} = 5$

11. $\frac{x - 5}{6} = \frac{x}{4} - 1$