questions here!

## 5 Introduction

## Part I:

| Introduction | Rational for Learning Work |
| :--- | :--- | Problems

In this first part of the lesson, you will learn how to solve work problems. Work problems are those that involve the speeds of people and/or machines. We will teach you how to calculate how long it will take for multiple workers to perform a job. This lesson is broken down into three sections:

- Multiple Jobs Problems
- Delayed Start Work Problems
- Unknown Rate Work Problems

In order to delve into work problems, it should be understood why it is we study them:

Reasons to Learn Work Problems

- Demonstrates Need to Understand Rational Expressions
- Learn How to Solve Rational Equations
- Hone Problem Solving Skills and Mathematical Reasoning
$\qquad$
- Solve Real World Time-Management Problems


## Work Basics

If Bridget needs three hours to paint a room, how much work will she get done in an hour of time?

To solve this problem, we will have to divide a single job into parts. We will divide the job into three equal parts, which will be one part for each hour being worked as follows:

| Hour 1 | Hour 2 | Hour 3 |
| :---: | :---: | :---: |
| $\frac{1}{3}$ | $\frac{1}{3}$ | $\frac{1}{3}$ |

The total job can be represented as being a single job, which translates mathematically to a "1." Consequently, each hour of the job must constitute a third of the job, assuming Bridget works at the same speed throughout the entire job.

This is appropriate because $1 / 3+1 / 3+1 / 3=1$.
Now, if the question was changed so that it took five hours to do the job, we would see... $1 / 5+1 / 5+1 / 5+1 / 5+1 / 5=1$.

To fully understand this relationship, we should realize a three-hour job requires $1 / 3$ of the work to be completed per hour. A five-hour job demands that $1 / 5$ of the job gets handled per hour. Using mathematical terms, this relationship is called the reciprocal. So, a seven-hour job would translate to $1 / 7$ of the work completed per hour.

To solve work-rate problems it is helpful to use a variant of distance equals rate times time. Specifically:

$$
Q=r t
$$

In this formula $Q$ is the quantity or amount of work done, $r$ is the rate of work and $t$ is the time worked.

EX 1: If a machine can produce $2 \frac{1}{2}$ parts per minute then in:
4 minutes, it can produce
40 minutes, it can produce
$2 / 5$ minutes, it can produce
EX 2: If Paul can inventory a small stockroom in 2 hours, then his rate of work is stockrooms per hour.

## Part II: Solving Work Rate Problems

In most cases you will be asked to find out how long it will take to perform a task if more than one worker does a portion of the task. To make the problems easier to model you will assume the workers do not get in each other's way or find a more efficient way to work as partners. Each will do his or her own part.

What does the expression, "many hands make like work" mean?

If Karen works on a puzzle alone it would take her 7 hours to complete and if Susan works on the puzzle alone it would take her 4 hours to complete the puzzle. How long would it take them to work together?

|  | Rate | Time | Quantity Completed |
| :--- | :--- | :--- | :--- |
|  |  |  |  |
|  |  |  |  |
| Total Number of |  |  |  |

Karen's Portion of Puzzle + Susan's Portion of the Inventory $=$ Total Number of puzzles

Paul can complete an inventory in 2 hours. Alia can complete the same inventory in 3 hours. If they work together, how long will it take to complete 3 inventories?

|  | Rate | Time | Quantity <br> Completed |
| :--- | :--- | :--- | :---: |
|  |  |  |  |
|  |  |  |  |
| Total Number of |  |  |  |

$$
\begin{aligned}
& \text { Paul's Portion } \\
& \text { of the Inventory }
\end{aligned} \text { 伎 } \begin{gathered}
\text { Alia's Portion } \\
\text { of the Inventory }
\end{gathered}=\begin{gathered}
\text { Inventories }
\end{gathered}
$$

Sometimes one of the rates is essentially negative because one person or agent undoes the work of another.

Leto's department can deliver enough parts to fill a storeroom in 8 hours. Jessica's department uses the part to build another product. Her department can deplete a full storeroom in 12 hours. Starting with an empty storeroom, how long will it take Leto's area to fill the storeroom while Jessica's department is working?

|  | Rate | Time | Quantity <br> Completed |
| :--- | :--- | :--- | :---: |
|  |  |  |  |
|  |  |  |  |
| Total Number of |  |  |  |

How would the example above change if the storeroom were half-full when both departments started working?

## Delayed Start Work Problems

Derik owns a pool and wants to fill it. His hose will take 15 hours to fill the pool. His neighbor's hose will take 12 hours to fill the pool. If Derik starts to fill his pool with his hose and then two hours later also uses his neighbor's hose, how long will it take to fill his pool.

|  | Rate | Time | Quantity <br> Completed |
| :--- | :--- | :--- | :---: |
|  |  |  |  |
|  |  |  |  |
| Total Number of |  |  |  |

A garden hose can fill a swimming pool with water in 15 hours.
A larger hose can do the job in 10 hours.
A fire hose can do the job in 6 hours.


|  | Rate | Time | Quantity <br> Completed |
| :--- | :--- | :--- | :---: |
|  |  |  |  |
|  |  |  |  |
|  |  |  |  |
| Total Number of |  |  |  |

## Unknown Rate Work Problems

It takes Tom 4 hours to build a fence.
If he hires Jack to help him, together they can do the job in just 3 hours. If Jack built the same fence alone, how long would it take him?

|  | Rate | Time | Quantity <br> Completed |
| :--- | :--- | :--- | :--- |
|  |  |  |  |
|  |  |  |  |
| Total Number of |  |  |  |

## SUMMARY:



### 4.5 Work Rate Problem Set

## Solve the "Work" problems by setting up a table.

EX 1: Paul can complete an inventory in 2 hours. Alia can complete the same inventory in 3
hours. If they work together, how long will it take to complete one inventory?

EX 2: Suppose one painter can paint the entire house in twelve hours, and the second painter takes eight hours. How long would it take the two painters together to paint the house?

EX 3: Krystal can wax a floor in 16 minutes. One day her friend Perry helped her, and it only took 5.76 minutes. How long would it take Perry to do it alone?
4. A cold-water pipe can fill a swimming pool in 10 hours, and a hot-water pipe can fill the pool in 15 hours. How long will it take to fill the pool if both pipes are left open?
5. Gurney can mow the lawn in $3 \frac{1}{2}$ hours. Moneo can mow the lawn in $4 \frac{1}{4}$ hours. How long will it take them to mow the lawn if they work together?
6. Siona can wash and wax a car in $1 \frac{1}{2}$ hours. Vladimir can do the same job in 2 hours. How long will it take them to do the job together?
7. An inlet pipe can fill a barrel of vinegar in 8 hours and an outlet pipe can empty it in 12 hours. How long will it take to fill the barrel if both pipes are left open?
8. A tank can be filled by an inlet pipe in 12 hours and emptied by a drain pipe in 16 hours. How long will it take to fill the tank if both pipes are left open?
9. Mr. and Mrs. Corrino can complete a job in 5 hours. If Mr. Corrino works twice as long as Mrs. Corrino if each does the job alone, how long does it take Mrs. Corrino to complete the job alone?
10. Tharthar and Shoab can complete a job in 3 hours working together. If Shoab works three times as long as Tharthat if each does the job alone, how long would it take Shoab to complete the job alone?
11. One person can clean the house in 5 hours. If a second person helps to clean the house the job can be done in 2 hours. How long should it take the second person to clean the house?
12. Stilgar can mow the lawn in 3 hours. If Stilgar and Jamis work together the lawn can be mowed in 2 hours. How long would it take Jamis to mow the lawn alone?

EX 13:
One printing press can produce a newspaper in 6 hours running alone. A second press could produce the paper in 9 hours running alone. A third press could produce the paper in 12 hours working alone. A late-breaking headline caused the newspaper to change the paper later than usual. If all three presses are run, how long will it take to produce the newspaper?

EX 14: One pipe can fill a pool 1.25 times as fast as a second pipe. When both pipes are opened, they fill the pool in five hours. How long would it take to fill the pool if only the slower pipe is used?

EX 15: Maria can do a certain job in 10 minutes. George can do the same job in 7 minutes. Maria and George work together to complete the same job. However, Maria starts working 2 minutes after George starts to work. How many minutes will it take for them to complete the job once George starts to work?

EX 16: Shawna can do a certain job in 10 months. Mary can do the same job in 5 months. Shawna and Mary work together to complete the same job. However, Shawna starts working 4 months after Mary starts to work. How many months will it take for them to complete the job once Mary starts to work?

EX 17: Lee can do a certain job in 6 months. David can do the same job in 5 months. If Lee and David work together to complete 3 of the same jobs, how many months will it take for them to complete these 3 jobs?

EX 18: Working alone, Carlos can oil the lanes in a bowling alley in five hours. Jenny can oil the same lanes in nine hours. If they worked together how long would it take them to oil 16 lanes?

EX 19: Working alone, it takes Asanji eight hours to dig a 10 ft by 10 ft hole. Brenda can dig the same hole in nine hours. How long would it take them to 6 dig a 10 ft by 10 ft holes if they worked together?

EX 20: Write your own "work" problem and solve it.

| 1. $54-6 \div 2+6=$ ? <br> A. 6 <br> B. 24 <br> C. 27 <br> D. 30 <br> E. 57 | 2. The lowest temperature on a winter morning was $-8^{\circ} \mathrm{F}$. Later that same day the temperature reached a high of $24^{\circ} \mathrm{F}$. By how many degrees Fahrenheit did the temperature increase? <br> A. $3^{\circ}$ <br> B. $8^{\circ}$ <br> C. $16^{\circ}$ <br> D. $24^{\circ}$ <br> E. $32^{\circ}$ |
| :---: | :---: |
| 3. Saying that $4<x<9$ is equivalent to saying what about x ? <br> A. $0<x<5$ <br> B. $0<x<65$ <br> C. $2<x<3$ <br> D. $4<x<9$ | 4. If the total cost of x apples is b cents, what is a general formula for the cost, in cents, of $y$ <br> A. $\frac{b}{x y}$ apples? <br> B. $\quad \frac{x}{b y}$ <br> C. $\frac{x y}{b}$ <br> D. $\frac{b y}{x}$ <br> E. $\frac{b x}{y}$ |

5. Of every 40 games a baseball team plays, it loses 12 games. What is the ratio of the team's losses to wins?
A. 3:10
B. 7:10
C. $3: 7$
D. $7: 3$
E. 10:3
