### [PACKET 5.1: POLYGON X-SUM THEOREM]



# **Polygon Angle Exploration**

We all know that the sum of the angles of a triangle equal 180°. What about a quadrilateral? A pentagon? We can answer this question by looking for a pattern:

# of Sides	Picture	# of Δ's	sum of all interior angles
3			
4			
5			
6			
	:		
n			

# Polygon Angle-Sum Theorem:

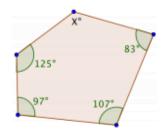
The sum of the measures of the interior angles of a polygon is	
where n is the number of sides of the polygon.	

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Examples:

Find x.





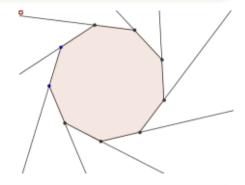
## What about Regular Polygons?

Because each polygon with n sides also has n angles of equal measure, you can divide the sum of the angles by n to find the measure of one angle.

## Regular Polygon Interior Angle Theorem:

The measure of ONE of the interior angles of a regular polygon is \_\_\_\_\_\_ where n is the number of sides of the polygon.

#### What about Exterior Angles?



#### Polygon Exterior Angle-Sum Theorem:

The sum of the measures of the exterior angles of ANY polygon is 360°.

### [PACKET 5.1: POLYGON X-SUM THEOREM]

#### Examples:

Find the measure of one interior angle in each regular polygon.

1



2. regular 15-gon

Find the measure of one exterior angle in each regular polygon.

3.



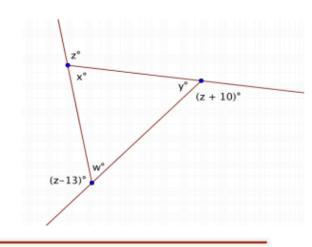
regular 13-gon

Find the sum of the interior angles for each polygon.

5. a decagon

6.

Find the value of each variable:



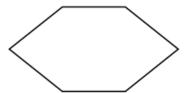
Now, Summarize your notes here!

#### **INTRO ACTIVITIES**

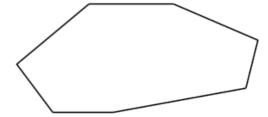
You can use what you know about the sum of the interior angle measures of a triangle to calculate the sum of the interior angle measures of a quadrilateral, as well as those of other polygons.

**DIAGONAL:** A line segment that connects two opposite vertices of a polygon.

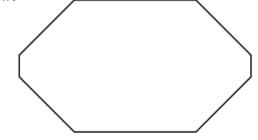
- Draw a quadrilateral. Select <u>ONE</u> vertex and draw all the diagonals that connect to that vertex.
- 2) In #1, notice that the diagonal divides the quadrilateral into two triangles. Since you know the sum of the measures of the interior angles in each triangle, you can determine the sum of the measures of the four interior angles of the quadrilateral. What is it? Explain your response.
- 3) Draw a pentagon. Select <u>ONE</u> vertex and draw all the diagonals that connect to that vertex. Then complete the "Quadrilateral" line in the table below:
- 4) Here is a hexagon. Select <u>ONE</u> vertex and draw all of the diagonals that connect to that vertex. Then complete the "Hexagon" line in the table below:



5) Here is a heptagon. Select <u>ONE</u> vertex and draw all of the diagonals that connect to that vertex. Then complete the "Heptagon" line in the table below:



6) Here is an octagon. Select <u>ONE</u> vertex and draw all of the diagonals that connect to that vertex. Then complete the "Octagon" line in the table below:



- 7) In the table, there is a line for a "13-gon." A 13-gon is a polygon with 13 sides. Similarly, a "21-gon" is a polygon with 21 sides. Complete the 13-gon and 21-gon lines in the table.
- 8) A follows from #7, an n-gon is a polygon with n sides. Complete the last line of the table in terms of n.

Name of polygon	Number of sides	Number of diagonals from one vertex	Number of triangles formed	Sum of interior angles in a triangle	Sum of interior angles in this polygon
Triangle	3	0	1	180°	180°
Quadrilateral					
Pentagon					
Hexagon					
Heptagon					
Octagon					
:	:	:	:		:
13-gon					
:	:	:	:		:
21-gon					
:	:	:	÷		:
n-gon	n				

<sup>9)</sup> Explain in complete sentences how to calculate the sum of the interior angles in any given polygon.

<sup>10)</sup> Go to <a href="https://www.urquhartmath.weebly.com">www.urquhartmath.weebly.com</a> and check out the walk the polygon activity applet. Then, answer the question: "What is the sum of all the exterior angles of an convex polygon?"

#### **PRACTICE PROBLEMS**

Find the sum of the interior angles of the given polygons.

Nonagon

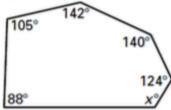
15-gon

3) 30-gon

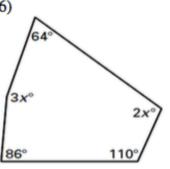
4) Dodecagon

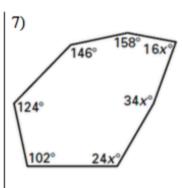
Find the value of x.

5)



6)





Recall that a REGULAR POLYGON is a polygon that is equilateral and equiangular.

Given the number of sides of a regular polygon, find the measure of each interior angle.

8) 10

9) 6

10) 18

Given the measure of each interior angle of a regular polygon, find how many sides the polygon has. 11) 108° 12) 140°

For the given regular polygons, is measure of each exterior angle.     a) Nonagon		find the sum of the interior angles, the measure of each interior  b) 11-gon			nterior angle, the
Sum:	Interior:	Exterior:	Sum:	Interior:	Exterior:

Sum:	Interior:	Exterior:	Sum:	Interior:	Exterior:

d) 16-gon

- 17) Given the measure of each angle of a regular polygon, find the number of sides of the polygon.
  - a) Exterior = 12°

c) Octagon

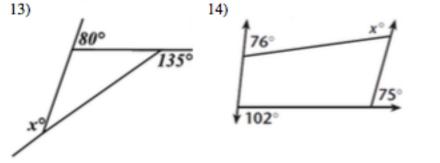
b) Exterior = 60°

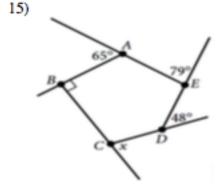
c) Exterior = 24°

- d) Interior = 162°
- e) Interior = 108°

f) Interior = 144°

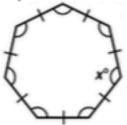
For each figure, a) find the value of the variable and b) find the sum of the measures of the exterior angles.

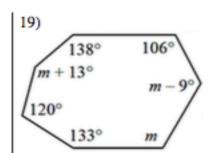


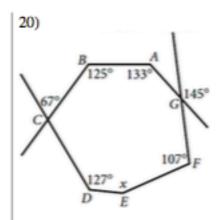


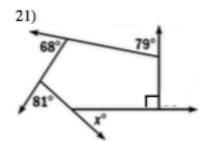
Find the value of each variable.

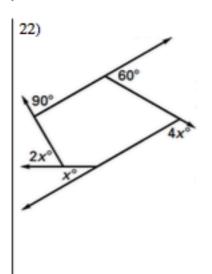


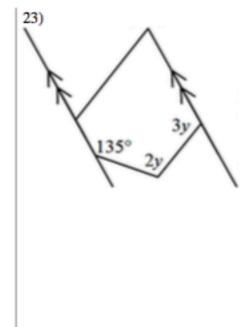




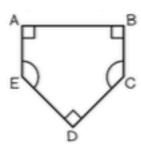




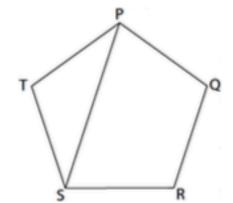




24) <u>BASEBALL:</u> A home plate marker for a softball field is a pentagon. Three of the interior angles of the pentagon are right angles. The remaining two interior angles are congruent. What is the measure of each angle?

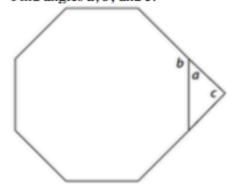


- 25) PQRST is a regular pentagon.
  - a) Find m∠STP. Explain.
- b) What kind of triangle is ASTP?

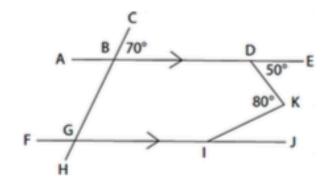


- c) Find m∠TSP . Explain
- d) Find m∠PSR. Explain

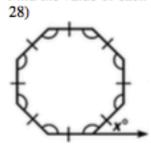
26) Two sides of this regular octagon have been extended to make a triangle on one of the sides. Find angles a, b, and c.

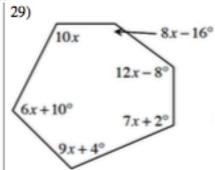


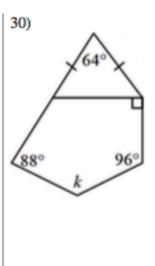
 Calculate the size of angle GIK, giving full reasons for your answer.

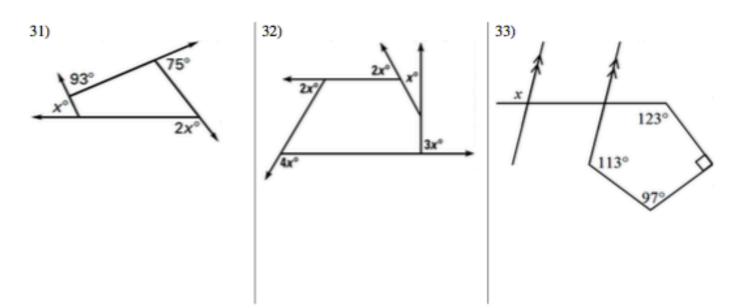


Find the value of each variable.





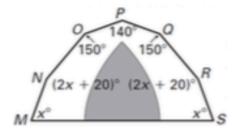




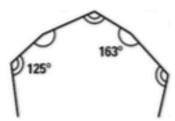
Determine the number of sides of the regular polygon described.

- 34) The measure of one interior angle is twice the measure of the exterior angle.
- 35) The measure of one interior angle is half the measure of the exterior angle.

36) The front view of a camping tent is shown. Find the value of x and determine the measure of each angle.



37) Part of an n-gon is shown. The pattern of congruent angles continues around the polygon. Find the value of n.



- 38) <u>POSSIBLE POLYGONS</u> Would it be possible for a regular polygon to have interior angles with the angle measure described? Explain.
  - a) 150°

b) 72°

c) 135°

d) 18°

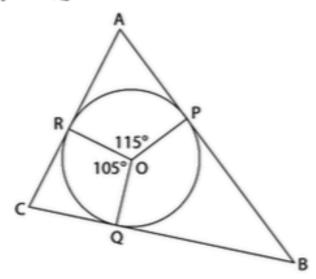
39) The sides of the AABC are tangents that touch a circle at points P, Q and R. O is the center of the circle.

a) Find m∠ARO and m∠APO. Explain.

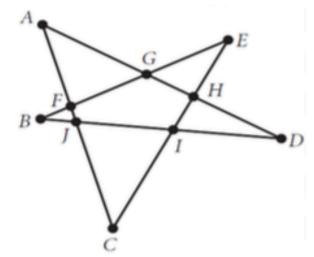






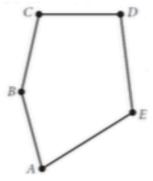


- e) Do the three angles of the triangle have the total you expect?
- 40) Given m∠A = 45°, m∠JFG = 100°, m∠FJI = 112°,  $m\angle GHI = 91^{\circ}$ , and  $m\angle C = 44^{\circ}$ , find the following.
- a) m∠B \_\_\_\_\_
- d) m∠HIJ \_\_\_\_\_
- b) m∠FGH \_\_\_\_\_
- e) m∠D \_\_\_\_\_
- c) m∠DHI \_\_\_\_\_\_ f) m∠E \_\_\_\_\_

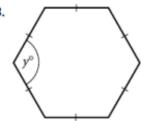


41) In the figure at the right,  $m \angle A = 7x + 6y$ ,  $m \angle B = 38y$ ,  $m \angle C = 13x + 3y$ ,  $m \angle D = 19x - 9y$ , and  $m \angle E = 17x - y$ . If  $m \angle A = 73^{\circ}$ ,  $m \angle C = 103^{\circ}$ , find the indicated measures.

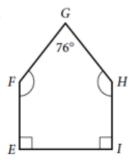
- 5. x \_\_\_\_\_\_ 6. y \_\_\_\_\_
- 7. m∠B \_\_\_\_\_\_\_ 8. m∠D \_\_\_\_\_
- 9. m∠E \_\_\_\_\_

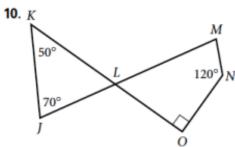


8.



9.





Solve each equation	for x!
	,

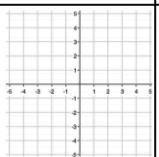
1. 
$$-2x - 3 = 5x - 3$$

4.

 $(x^2 + 6x + 8)$ 

5. Graph the equation:

$$y = 2 - x$$



6. Graph the equation:

