5.1 Triangle Congruence Postulates

**Congruent Figures:** Figures that are the same shape and size.

- All corresponding angles are congruent
- All corresponding sides are congruent

How are congruent figures different from similar figures? How are they the same?

______________________________________________________________

______________________________________________________________

______________________________________________________________

**CPCTC:** “All corresponding parts of congruent figures are congruent”

**Intro Activity:** (AngLegs)

**Lawrence the Laborer Story**

Lawrence works for a company that makes roof trusses, triangular pieces that support simple roofs. His job is to ensure that each and every roof truss that is made at the company is exactly the same size as all the others.

**Version A:** Because Lawrence is so lazy and likes to avoid as much work as possible, he wants to find the easiest way possible to do his job.

**Version B:** Because Lawrence is so lazy a no-nonsense person and likes to avoid as much unnecessary work as possible, he wants to find the easiest most efficient way possible to do his job.

How do you think his boss should view Lawrence?

______________________________________________________________

______________________________________________________________
Help Lawrence find the easiest most productive way to show that two triangular trusses are equal in measure with as little work wasted time as possible.

To do this test to see if the following 3-measurement shortcuts work, do they prove the two triangles are in fact congruent?

<table>
<thead>
<tr>
<th>SIDE-SIDE-SIDE</th>
<th>ANGLE-ANGLE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Δ ABC: Attach 1 purple to 1 red to 1 yellow.</td>
<td>Δ ABC: Attach 3 blue sticks together.</td>
</tr>
<tr>
<td>Δ DEF: Attach 1 red to 1 purple to 1 yellow.</td>
<td>Δ DEF: Attach 3 purple sticks together.</td>
</tr>
<tr>
<td></td>
<td>What is the degree of all the corresponding angles?</td>
</tr>
<tr>
<td></td>
<td>_________</td>
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</tbody>
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<tr>
<th>SIDE-ANGLE-SIDE</th>
<th>ANGLE-SIDE-ANGLE</th>
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<tbody>
<tr>
<td>Δ ABC: Attach a yellow stick to a 95° angle to a</td>
<td>Δ ABC: Attach a _____ ° angle to red stick to a</td>
</tr>
<tr>
<td>green stick. (Angle in middle)</td>
<td>_____ ° angle. (Side in middle)</td>
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<tr>
<th>ANGLE-SIDE-SIDE</th>
<th>HYPOTENUSE-LEG</th>
</tr>
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<tbody>
<tr>
<td>Δ ABC: Draw a 30° angle. Attach the 30° angle to a</td>
<td>Δ ABC: Attach a 90° angle to a purple stick to a</td>
</tr>
<tr>
<td>blue stick to a purple stick. (Blue side in middle)</td>
<td>blue stick</td>
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<tr>
<td>Δ ABC: 35° angle (Draw this out on a sheet of paper), 110° angle, &amp; 1 yellow stick</td>
</tr>
<tr>
<td>Δ DEF: 35° angle (Draw this out on a sheet of paper), 110° angle, &amp; 1 yellow stick</td>
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</tbody>
</table>
What is the purpose of this lesson?

To prove that two triangles are congruent using relationships between corresponding angles and sides between the two triangles.

You must have a combination of ___ corresponding angles and side to prove two triangles are congruent.

5 Triangle Congruence Postulates
- Side-Side-Side (SSS)
- Side-Angle-Side (SAS)
- Angle-Side-Angle (ASA)
- Angle-Angle-Side (AAS)
- HL (Hypotenuse-Leg)

otherwise known as RHS
(Right-angle-Hypotenuse-Side)

Side-Side-Side (SSS) Postulate

If the sides of one triangles are congruent to the sides of another triangle, then the two triangles are congruent by the SSS postulate.

Side-Angle-Side (SAS) Postulate

If two sides and the included angle of one triangle are congruent to two sides and the included angle of another triangle, then the two triangles are congruent by the SAS postulate.

Angle-Side-Angle (ASA) Postulate

If two angles and the included side of one triangle are congruent to two angles and the included side of another triangle, then the two triangles are congruent by the ASA postulate.
ASA versus AAS Postulates

**Angle-Side-Angle (ASA)**

\[ \triangle ABC \cong \triangle DEF \]
by ASA Postulate

**Angle-Angle-Side (AAS)**

\[ \triangle ABC \cong \triangle DEF \]
by AAS Postulate

Hypotenuse-Leg (HL) or Right-angle-Hypotenuse-Side (RHS) Postulate

If two right triangles have congruent hypotenuses and congruent corresponding sides, then the two triangles are congruent by the HL or RHS postulate.

\[ \star \text{ONLY FOR RIGHT TRIANGLES!!!} \star \]

Angle-Angle-Angle (AAA) Postulate

You can't prove two triangles are congruent to each other if you only compare their angle relationships.

TRY ON YOUR OWN: Then check answers with video 1 and video 2

State whether these pairs of triangles are congruent by SSS, SAS, ASA, AAS, or HL postulates. If none of these methods work, write None. None indicates that the triangles aren't congruent.

Ex. 1)  

Ex. 2)  

Ex. 3)
5.1 Problem Set

State if the two triangles are congruent. If they are, state why.

1. 

[Diagram of two congruent triangles]

2. 

[Diagram of two triangles labeled ΔDEG, ΔFGE]
5. GIVEN: $D$ is the midpoint of $AC$.

6.

7.

8.

9.

10.

11.

12.
21. Explain how you can prove that the indicated triangles are congruent using the given postulate or theorem.

a. \( \triangle ABE \cong \triangle CDE \) by SAS

b. \( \triangle ABE \cong \triangle CDE \) by ASA

c. \( \triangle ABE \cong \triangle CDE \) by AAS
<table>
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<tr>
<th>SOLVE</th>
<th>GRAPH</th>
<th>MULTIPLY</th>
</tr>
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<tbody>
<tr>
<td>(5 - 2(3x - 4) = -7)</td>
<td>(y = -x)</td>
<td>((5x - 3)(2x + 3))</td>
</tr>
</tbody>
</table>

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<th>SOLVE</th>
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<th>FACTOR</th>
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<tbody>
<tr>
<td>(\frac{2x - 1}{6} = \frac{x}{4})</td>
<td>(y = \frac{2}{3}x)</td>
<td>(x^2 - 10x - 24)</td>
</tr>
</tbody>
</table>

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**ACT Application Problems**

1. What postulate or theorem would you use to prove \(\triangle ABC \cong \triangle FED\)?
   - A. ASA
   - B. AAS
   - C. SAS
   - D. SSS
   - E. HL
   - F. None of the above.

2. What information is needed to prove that \(\triangle WXZ \cong \triangle YXZ\) by the SAS Congruence Postulate?
   - A. \(\overline{WZ} \cong \overline{YZ}\)
   - B. \(\angle WXZ \cong \angle XYZ\)
   - C. \(\angle WXZ \cong \angle YXZ\)
   - D. None of the above.

3. What postulate or theorem would you use to prove \(\triangle ABC \cong \triangle FEC\)?
   - A. ASA
   - B. AAS
   - C. SAS
   - D. SSS
   - E. HL
   - F. None of the above.
4. What postulate or theorem would you use to prove $\triangle GHI \cong \triangle KJI$?

A. ASA  
B. AAS  
C. SAS  
D. SSS  
E. HL  
F. None of the above.

5. What postulate or theorem would you use to prove $\triangle MNO \cong \triangle PQR$?

A. ASA  
B. AAS  
C. SAS  
D. SSS  
E. HL  
F. None of the above.

6. What postulate or theorem would you use to prove $\triangle OML \cong \triangle OMN$?

A. ASA  
B. AAS  
C. SAS  
D. SSS  
E. HL  
F. None of the above.

7. State the third congruence that is needed to prove that $\triangle DEF \cong \triangle MNO$ given that $DE \cong MN$ and $\angle M \cong \angle D$ using the SAS Congruence Postulate.

A. $DF \cong MO$  
B. $\angle E \cong \angle N$  
C. $EF \cong NO$  
D. None of the above.

8. State the third congruence that is needed to prove that $\triangle DEF \cong \triangle MNO$ given that $FE \cong ON$ and $\angle F \cong \angle O$ using the AAS Congruence Theorem.

A. $\angle D \cong \angle M$  
B. $DF \cong MO$  
C. $\angle E \cong \angle N$  
D. None of the above.
9. State the third congruence that is needed to prove that $\triangle DEF \cong \triangle MNO$ given that $\overline{FE} \cong \overline{ON}$ and $\angle F \cong \angle O$ using the ASA Congruence Postulate.

A. $\angle D \cong \angle M$
B. $\overline{DF} \cong \overline{MO}$
C. $\angle E \cong \angle N$
D. None of the above.

10. Which of these statements could NOT be the third congruence that is needed to prove that $\triangle ABC \cong \triangle XYZ$ given that $\angle A \cong \angle X$ and $\angle B \cong \angle Y$ using the AAS Congruence Postulate?

A. $\overline{BC} \cong \overline{YZ}$
B. $\overline{AB} \cong \overline{XY}$
C. $\overline{AC} \cong \overline{XZ}$
D. None of the above.

11. State the third congruence that is needed to prove that $\triangle ABC \cong \triangle XYZ$ given that $\angle A \cong \angle X$ and $\overline{AB} \cong \overline{XY}$ using the ASA Congruence Postulate.

A. $\angle C \cong \angle Z$
B. $\overline{AC} \cong \overline{XZ}$
C. $\angle B \cong \angle Y$
D. None of the above.

12. State the third congruence that is needed to prove that $\triangle ABC \cong \triangle XYZ$ given that $\overline{BC} \cong \overline{YZ}$ and $\angle C \cong \angle Z$ using the SAS Congruence Postulate.

A. $\overline{AC} \cong \overline{XZ}$
B. $\angle X \cong \angle A$
C. $\overline{AB} \cong \overline{XY}$
D. None of the above.

13. What theorem or postulate would you use to prove that the triangles are congruent?

A. ASA
B. AAS
C. SAS
D. SSS
E. HL
F. None of the above.
14. What theorem or postulate would you use to prove that the triangles are congruent?

A. ASA
B. AAS
C. SAS
D. SSS
E. HL
F. None of the above.

15. What theorem or postulate would you use to prove that the triangles are congruent?

A. ASA
B. AAS
C. SAS
D. SSS
E. HL
F. None of the above.

16. What theorem or postulate would you use to prove that the triangles are congruent?

A. ASA
B. AAS
C. SAS
D. SSS
E. HL
F. None of the above.

FLASHBACK PROBLEMS:

17. Find the value of $x + y$.

A. 10
B. 71
C. 81
D. 109
E. None of the above.

18. Find the length of a side.

A. 4
B. 11
C. 33
D. None of the above.

19. Find the measure of the vertex angle.

A. 25
B. 52
C. 76
D. None of the above.

20. Find the perimeter of the triangle.

A. 2
B. 6
C. 8
D. 28
E. None of the above.