Quadrilateral Proofs – Packet #2

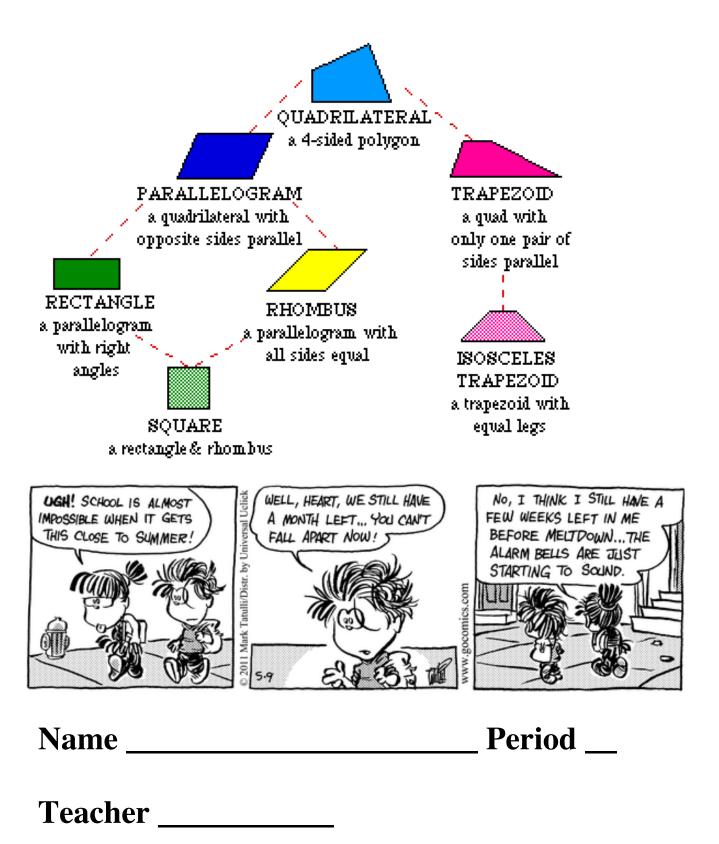


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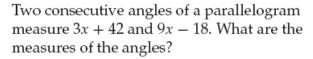
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Day 1 – Parallelograms

Warm – Up

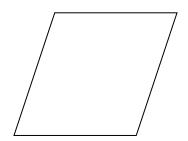




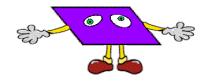
B 58.5, 31.5

C 39, 141

D 81, 99



Properties of the Parallelogram



Parallelogram

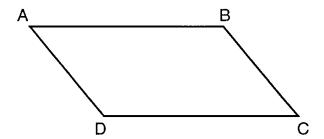
A parallelogram is a quadrilateral with two pairs of parallel sides. All parallelograms, such as $\Box FGHJ$, have the following properties.



Properties of Parallelograms		
$ \begin{array}{ccc} G & & & & & & & & \\ F & & & & & & \\ F & & & & & & \\ & & & & & & \\ & & & & & &$	Opposite angles are congruent.	∠F≅ ∠H ∠G≅ ∠J
	The diagonals bisect each other.	FP ≅ HP GP ≅ JP

Complete the statement and give the reason that justifies the statement.

Given: ABCD is a parallelogram

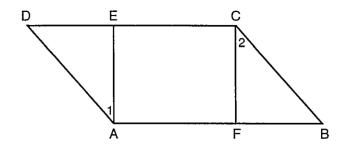


Statements	Reasons
a. $\overline{AB} \cong \underline{\hspace{1cm}}$ and $\overline{AD} \cong \underline{\hspace{1cm}}$	a.
b ≰A ≅ and ≰D ≅	b.
c. $\overline{AB} \parallel \underline{\hspace{1cm}}$ and $\overline{AD} \parallel \underline{\hspace{1cm}}$	c.
d. 4A <i>suppl</i> and	d.
4C Suppi and	
e. Draw \overline{AC} and \overline{BD} . (The lines intersect at E.)	e. Two Points Make a Line.
f. ∡BAC ≅ and ∡DAC ≅	f.
g. $\overline{AE} \cong \underline{\qquad}$ and $\overline{DE} \cong \underline{\qquad}$	g.

Proofs

Given: ☐ ABCD DE ≅ FB

Prove: a) ∆DEA ≅ ∆BFC b) ∠1 ≅ ∠2



STATEMENT

1. Parallelogram ABCD

2. AD ≅ ____

3. <u>∠D</u> ≅ ____ 4. DE ≅ FB

5. 6.

REASONS

1. Given

2. 3.

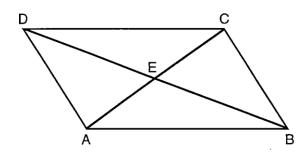
4. Given

5.

6.

Given: ABCD

Prove: △AEB ≅ △CED



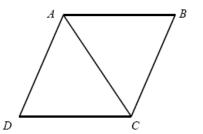
REASONS STATEMENT 1. Given 1. Parallelogram ABCD 2. 2. AB ≅ ____ 3. 3. AB || ____ 4. 4. ∠CAB ≅ ____ 5. 5. ∠AEB ≅ ____ 6. 6.

You Try It!

Given: □ABCD

Prove: $\Delta DAC \cong \Delta BCA$

(At most 6 steps! You may not need all 6!!!)

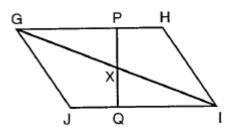


Statements Reasons

1	1
2	2
3	3
4	4
5	5
6	6

Given: GHU

Prove: $\overline{PX} \cong \overline{QX}$



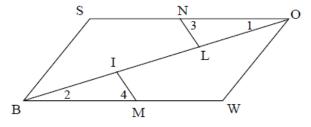
Statements	Reasons
1	1
2	2
3	3
4	4
5	5
6	6
7	7
8	8

You Try It!

Given: WOSB is a parallelogram

∡3≅ **∡**4 $\overline{MW} \cong \overline{SN}$

Prove: $IM \cong LN$



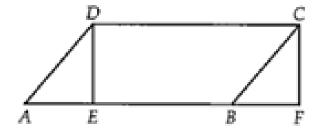
Statements

Reasons

Challenge

Given: \overrightarrow{ABCD} is a parallelogram. $\overrightarrow{DE} \perp \overrightarrow{AF}$. $\overrightarrow{CF} \perp \overrightarrow{AF}$.

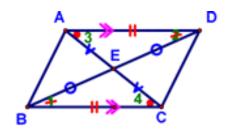
Prove: $\overline{DE} \cong \overline{CF}$.



SUMMARY

Given: Parallelogram ABCD

Prove: AC and DB bisect each other



Statements

- 1. Parallelogram ABCD
- 2. AD || BC
- A 3. ∠1 ≈ ∠2
- A 4. ∠3 ≈ ∠4
- S 5. AD ≈ BC
 - 6. △BEC ≃ △DEA
 - 7. DE ≈ BE
 - 8. AE ≃ CE
 - 9. AC and DB bisect each other

- 1. Given
- 2. Opposite sides of a parallelogram are ||

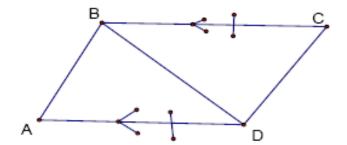
Reasons

- 3. // → AIA's ≅
- 4. // → AIA's ≅
- 5. Opposite sides of a parallelogram are =
- 6. ASA (3, 5, 4)
- CPCTC
- 8. CPCTC
- 9. 2 ≅ segs → segment bisector

Exit Ticket

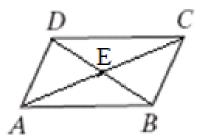
Identify the reason that proves $\triangle ABD \cong \triangle CDB$.

- A) SSS
- B) SAS
- C) ASA
- D) AAS



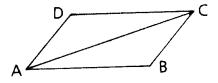
<u>HW - Day 1</u>

1) Given: ABCD is a parallelogram. Prove: $\triangle AEB \cong \triangle CED$



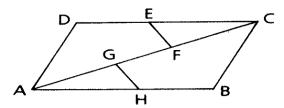
Statements Reasons

2) Given: \square ABCD (ABCD is a \square .) Conclusion: \triangle ABC \cong \triangle CDA



Statements Reasons

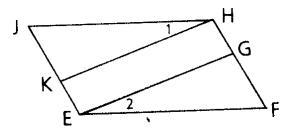
 $HB \cong DE$ Conclusion: $\overline{GH} \cong \overline{EF}$



Statement	Reason
1	1 Given
2	2 Opposite sides of a □ are .
3	3 ∥ lines ⇒ alt. int. ∠s ≅
4	4 Opposite sides of a □ are ≅.
5	5 Given
6	6 Subtraction Property(,)
7	7 Given
8	8 ASA (3, 6, 7) 9 CPCTC
9	9 CPCTC

4) Given: \square EFHJ, $\angle 1 \cong \angle 2$

Conclusion: $\overline{KH} \cong \overline{EG}$



Statements Reasons

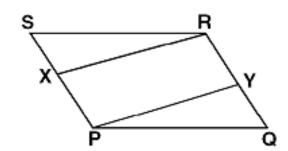
<u>Day 2 - Ways to prove a quadrilateral is a</u> <u>parallelogram</u>

Warm - Up

Given: ⊿PQRS

 $\overline{XP} \cong \overline{RY}$

Prove: $\overline{XR} \cong \overline{YP}$



Statements	Reasons
1.	1. Given
$2. \overline{SP} \cong \overline{QR}$	2.
≅ (S)	
3. (S)	3.
$4. \ \not \Delta S \cong \ \not \Delta Q \qquad (A)$	4.
5.	5.
$6. \ \overline{XR} \cong \overline{YP}$	6.

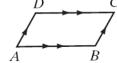
You can show that a quadrilateral is a parallelogram by using any of the conditions listed below.

Conditions for Parallelograms

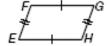
- Both pairs of opposite sides are parallel (definition).
- One pair of opposite sides is parallel and congruent.
- Both pairs of opposite sides are congruent.
- Both pairs of opposite angles are congruent.
- The diagonals bisect each other.
- One angle is supplementary to both its consecutive angles.

Determine whether each quadrilateral must be a parallelogram. Justify your answer.

a.



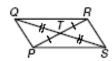
b.



c



А



e

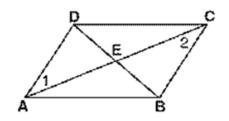


f.

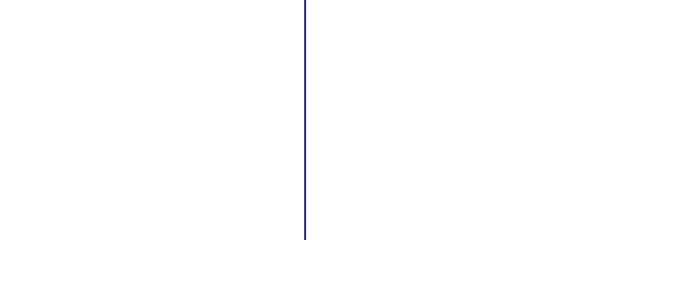


1. Given: \overline{DB} bisects \overline{AC} $\angle 1 \cong \angle 2$

Prove: ABCD is a parallelogram



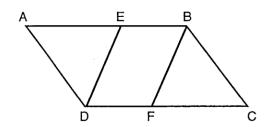
Statements Reasons



2.

Given: \triangle ABCD AE \cong CF

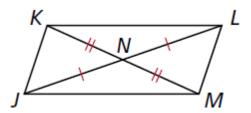
Prove: ZEBFD



STATEMENT	REASONS
1. AE ≅ CF, ∠ ABCD 2. ∠A ≅ 3. AD ≅ 4. Δ ≅ Δ 5. ED ≅ 6. EB ≅ 7. EBFD is a	1. 2. 3. 4. 5. 6. 7.

You Try It!

Given: \overline{JL} and \overline{KM} bisect each other. **Prove:** JKLM is a parallelogram.



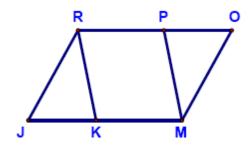
Statements	Reasons

Challenge

Given: RKMP is a □

∠JRK ≃ ∠PMO

Prove: RJMO is a □



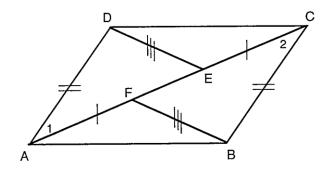
Reasons

Statements

SUMMARY

GIVEN: $\overline{AD} \cong \overline{BC}$ $\overline{AF} \cong \overline{EC}$ $\overline{DE} \cong \overline{FB}$

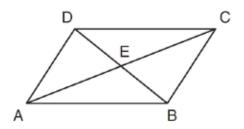
PROVE: ABCD is a 🗁



STATEMENTS	1	REASONS
1. AD ≅ BC (s DE ≅ FB (s	≅ s) ≅ s)	1. Given
2. ĀF ≅ ĒC		2. Given
3. AE ≅ CF (AF + FE = CE + FE) (s	≅ s)	3. Addition Postulate
4. ΔADE ≅ ΔCBF	!	4. s.s.s. ≅ s.s.s.
5. ∠1 ≅ ∠2		5. Corresponding parts of congruent triangles are congruent.
6. AD BC		If two lines are cut by a transversal and a pair of alternate interior angles are congruent, then the lines are parallel.
7. ABCD is a 🗁		 A quadilateral is a parallelogram if a pair of sides are both congruent and parallel. [Steps 1 and 6]
 4. ΔADE ≅ ΔCBF 5. ∠1 ≅ ∠2 6. AD BC 	≅ s)	 4. s.s.s. ≅ s.s.s. 5. Corresponding parts of congruent triangles are congruent. 6. If two lines are cut by a transversal and a pair of alternate interior angles are congruent, then the lines are parallel. 7. A quadilateral is a parallelogram if a pair of sides are both congruent and parallel.

Exit Ticket

In the diagram below, parallelogram ABCD has diagonals \overline{AC} and \overline{BD} that intersect at point E.



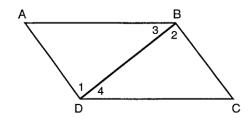
Which expression is not always true?

- ∠DAE ≅ ∠BCE
- ∠DEC ≅ ∠BEA
- 3) $\overline{AC} \cong \overline{DB}$
- 4) $\overline{DE} \cong \overline{EB}$

Day 2 - HW

Given: ∠1 ≅ ∠2
 ∠3 ≅ ∠4

Prove: ABCD

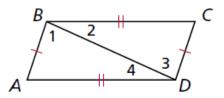


STATEMENT	REASONS
1. ∠1 ≅ ∠2	1.
2. BC	2.
3. ∠3 ≅ ∠4	3.
4. AB	4.
5. ABCD is a	l 5.

2. Given: $\overline{AB} \cong \overline{CD}$, $\overline{BC} \cong \overline{DA}$

Prove: *ABCD* is a parallelogram.

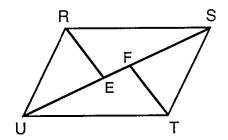
Proof:



Statements	Reasons
1. $\overline{AB} \cong \overline{CD}$, $\overline{BC} \cong \overline{DA}$	1. Given
2. $\overline{BD} \cong \overline{BD}$	2. a. ?
3. △ <i>DAB</i> ≅ b ?	3. c. <u>?</u>
4. ∠1 ≅ d. ? , ∠4 ≅ e. ?	4. CPCTC
5. $\overline{AB} \parallel \overline{CD}$, $\overline{BC} \parallel \overline{DA}$	5. f. <u>?</u>
6. ABCD is a parallelogram.	6. g. <u>?</u>

Given: $\angle REU \cong \angle TFS$ $UE \cong SF, RE \cong FT$ 3.

Prove: ARSTU



Statements	Reasons

Given: Quadrilateral ABCD

4. $\overline{AB} \cong \overline{CD}$ $\overline{AB} \parallel \overline{CD}$

Prove: ABCD is a parallelogram



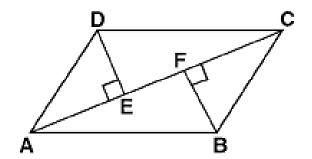
Statements Reasons

Day 3 – Proofs with Special Parallelograms

Warm - Up

Given: $\overline{DE} \perp \overline{AC}$ $\overline{BF} \perp \overline{AC}$ $\overline{AE} \cong \overline{FC}$ $\overline{DE} \cong \overline{FB}$

Prove: ABCD is a parallelogram



<u>Statements</u>	Reasons
1.	1. Given
2. 4 and 4 are right 4s	2.
2. ∠ and ∠ are right ∠s 3. ∠ ≅ ∠ (A)	3.
4. (S)	4. Given
5. (S)	5. Given
6. Δ ≅ Δ	6. (,,)
7.	7. CPCTC
8.	8. CPCTC
9. ABCD is a parallelogram	9.

Rectangles

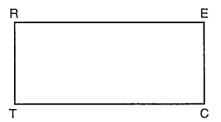
<u>Definition:</u> A **rectangle** is a parallelogram with one right angle.



Theorems Properties of Rectangles			
	THEOREM	HYPOTHESIS	CONCLUSION
6-4-1	If a quadrilateral is a rectangle, then it is a parallelogram. (rect. $\rightarrow \Box$)	$A \longrightarrow D$	ABCD is a parallelogram.
6-4-2	If a parallelogram is a rectangle, then its diagonals are congruent. (rect. → diags. ≅)	B C D	ĀC ≅ BD

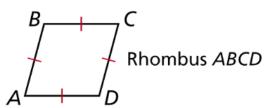
Complete the statement and give the reason that justifies the statement.

Given: Rectangle RECT



a line a line

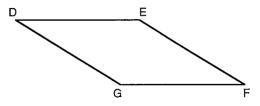
A **rhombus** is a quadrilateral with four congruent sides.



Theore	ms Properties of Rhombuses)	
	THEOREM	HYPOTHESIS	CONCLUSION
6-4-3	If a quadrilateral is a rhombus, then it is a parallelogram. (rhombus $\rightarrow \square$)	$A \xrightarrow{B} C$	ABCD is a parallelogram.
6-4-4	If a parallelogram is a rhombus, then its diagonals are perpendicular. (rhombus \rightarrow diags. \bot)	A D C	ĀC ⊥ BD
6-4-5	If a parallelogram is a rhombus, then each diagonal bisects a pair of opposite angles. (rhombus → each diag. bisects opp. ଛ)	A 87 65 D	∠1 ≅ ∠2 ∠3 ≅ ∠4 ∠5 ≅ ∠6 ∠7 ≅ ∠8

Complete the statement and give the reason that justifies the statement.

Given: Rhombus DEFG



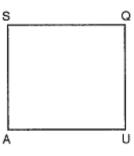
STATEMENT	REASONS
a. $\overrightarrow{DE} \cong \overrightarrow{EF} \cong \overrightarrow{FG} \cong \overrightarrow{GD}$	a.
 b. ∠D ≅ ∠F, ∠E ≅ ∠G c. Draw DF 	b. c. 2 points make a line
d. ∠GDF ≅ ∠EDF	d.
e. ∠EFD ≅ ∠GFD	e.
f. Draw EG (to intersect DF at X)	f. 2 points make a line
g. DF \	g.
h. $\overrightarrow{EX} \cong \underline{\hspace{1cm}}$ and $\overrightarrow{DX} \cong \underline{\hspace{1cm}}$	h.

A **<u>square</u>** is a quadrilateral with four right angles and four congruent sides. In the exercises, you will show that a square is a parallelogram, a rectangle, and a rhombus. So a square has the properties of all three.



Complete the statement and give the reason that justifies the statement.

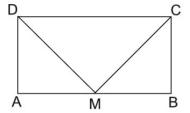
Given: Square SQUA



STATEMENT	REASONS	
a. ∠S is a angle b. SQ ≅ QU ≅ UA ≅ SA c. ∠S ≅ ∠Q ≅ ∠U ≅ ∠A d. Draw SU and QA e. SU ⊥ f. SU ≅ g. ∠ASU ≅ ∠QSU h. ∠SUA ≅ ∠QUS	a. b. c. d. 2 points make a line e. f. g. h.	
	ı	

Example 1: **Given:** ABCD is a rectangle M is midpoint of \overline{AB}

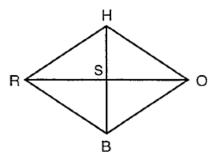
Prove: $\overline{DM} \cong \overline{CM}$



Statements Reasons

2) Given: Rhombus RHOB

Prove: ∠HSR ≅ ∠HSO



Statements

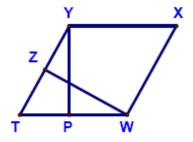
Reasons

Challenge

Given: YTWX is a □

YP ⊥ TW ZW ⊥ TY TP ≃ TZ

Prove: YTWX is a rhombus



Statements

Reasons

SUMMARY

Given: ABCD is a rhombus.

diagonals AC, BD

ΔAED ≈ ΔAEB ≈ Prove:

 $\triangle BEC = \triangle CED$

1 ABCD is a rhombus.

2 Diagonals AC, BD

 $3 \overline{AB} = \overline{BC} = \overline{CD} = \overline{DA}$

4 AC, BD ⊥ bis each other. 4 Diag of rhombus ⊥ bis

 $5 \overline{AE} \cong \overline{EC}, \overline{DE} \cong \overline{EB}$

6 △AED = △AEB =

 $\triangle BEC = \triangle CED$

1 Given

2 Given

3 All sides of a rhombus ≅.

each other.

5 Bis divides a seg into 2 ≅

segs.

6 SSS

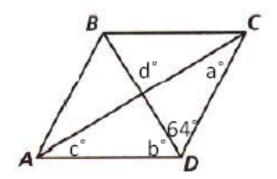
Given: MPRS $\overline{MO} \cong \overline{PO}$ Prove: AROS is isos. 1 MPRS 1 Given $2 \overline{MO} \cong \overline{PO}$ 2 Given $3 \overline{SM} \cong \overline{RP}$ 3 Opp sides ≡ in a . 4 ∠M is art ∠. 4 In a ____ all ∠s are rt ∠s. 5 LP is art L. 5 Same as 4 $6 \angle M \cong \angle P$ 6 All rt∠s are =. 7 △SMO ≈ △RPO 7 SAS $8 \overline{SO} \approx \overline{RO}$

8 CPCTC

9 An isos △ has 2 sides ≃.

Exit Ticket

Find the value of each variable in the rhombus.



9 △ROS is isos.

b =

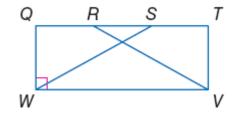
c = ____

d =

1. Given: QTVW is a rectangle.

 $\overline{QR}\cong \overline{ST}$

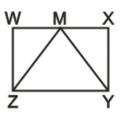
Prove: $\triangle SWQ \cong \triangle RVT$



Statements Reasons

2. Given: Rectangle WXYZ, M is the midpoint of \overline{WX} .

Prove: AZMY is isosceles.

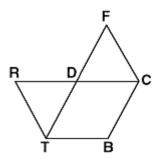


Statements Reasons

3. Given: TBCD is a rhombus

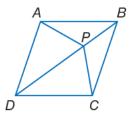
∠RTB ≃ ∠FCB

Prove: RD ≃ DF



Statements	Reasons
Statements	Rea

4. Write a two-column proof to prove that if ABCD is a rhombus with diagonal \overline{DB} , then $\overline{AP} \cong \overline{CP}$.



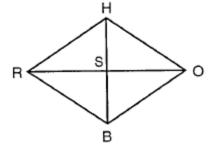
Statements	Reasons

Day 4 – Proofs with Trapezoids

Warm - Up

Given: $\overline{HS} \cong \overline{SB}, \overline{RS} \cong \overline{SO}$ $\overline{HR} \cong \overline{HO}$

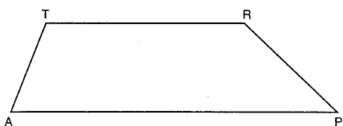
Prove: RHOB is a rhombus



Statements	Reasons
1.	1. Given
2. Δ ≅ Δ	2. (, ,)
Δ . Δ = Δ	
3. RHOB is a Parallelogram	3 of a quadrilateral
	bisect each other →
4. RHOB is a Rhombus	$4. \cong \underline{\hspace{1cm}}$ sides of a
	Parallelogram → Rhombus

Complete the statement and give the reason that justifies the statement.

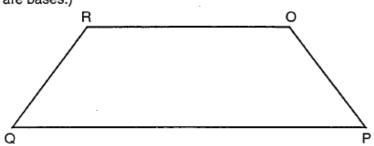
Given: Trapezoid TRPA (AP and TR are bases)



STATEMENT	REASONS	
a. TR	a.	
b. ∠T and are supplementary	b.	
c. ∠R and are supplementary	c.	

Complete the statement and give the reason that justifies the statement.

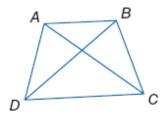
Given: Isosceles Trapezoid RQPO (OR and PQ are bases.)



REASONS
a.
b
c.
d.
e. 2 points make a line
f.
g.
h.

Example 1: Given: ABCD is an isosceles trapezoid.

Prove: $\triangle ADC \cong \triangle BCD$

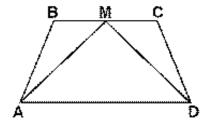


Statements	Reasons

Example 2: Given: ABCD is an isosceles trapezoid $\overline{\rm AD} \mid \mid \overline{\rm BC}$

M is the midpoint of $\overline{\text{BC}}$

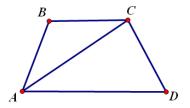
Prove: ΔMAD is isosceles



Statements Reasons

Challenge: Given: \overrightarrow{AC} bisects $\angle BAD$. $\overline{AB} \cong \overline{BC}$, $\overline{AB} \not\parallel \overline{CD}$

Prove: ABCD is a trapezoid.



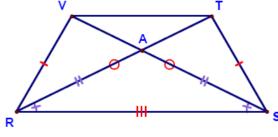
SUMMARY

Given: RVTS is an isos.

trapezoid with legs

VR & TS

Prove: △ARS is isosceles



Statements

Reasons

- 1. RVTS is an isos. trapezoid with legs VR & TS
- S 2. $\overline{VR} \equiv \overline{TS}$
- S 3. VS = TR
- S 4. RS ≡ RS
 - 5. △VRS ≡ △TSR
 - 6. ∠TRS ≡ ∠VSR
 - 7. AR = AS
 - 8. △ARS is isosceles

- 1. Given
- 2. Legs of an Isos. Trap are ≅
- 3. Diagonals of an Isos. Trap are ≅
- 4. Reflexive Property
- 5. SSS (2, 3, 4)
- 6. CPCTC
- 7. If \triangle , then \triangle
- 8. If \triangle , then Isos. \triangle

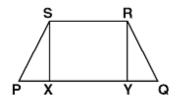
Exit Ticket

Isosceles trapezoid *ABCD* has diagonals \overline{AC} and \overline{BD} . If AC = 5x + 13 and BD = 11x - 5, what is the value of x?

- 1) 28
- 2) $10\frac{3}{4}$
- 3) 3
- 4) $\frac{1}{2}$

1. Given: PQRS is an isosceles trapezoid PQ | SR SX 上 PQ RY 上 PQ

 $\mathsf{Prove} \colon \angle \mathsf{PSX} \cong \angle \mathsf{QRY}$

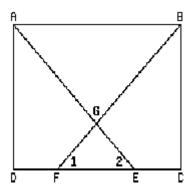


Statements Reasons

rectangle ABCD, $\overline{\text{DFEC}}$, $\overline{\text{AGE}}$, $\overline{\text{BGF}}$, and $\overline{\text{DF}} \cong \overline{\text{CE}}$. $2. \ {\rm Given:}$

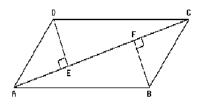
(a) ▲ADE ≃ ▲BCF Prove:

(b) ∠1 ≃ ∠2 (c) GF ≃ GE



Statements Reasons $3. \ \, \text{Given:} \quad \text{quadrilateral ABCD, diagonal $\overline{\mathsf{AEFC}}$, $\overline{\mathsf{DE}}$ \bot $\overline{\mathsf{AC}}$, $\overline{\mathsf{BF}}$ \bot $\overline{\mathsf{AC}}$, $\overline{\mathsf{AE}}$ \cong $\overline{\mathsf{CF}}$, and $\overline{\mathsf{DE}}$ \cong $\overline{\mathsf{BF}}$.}$

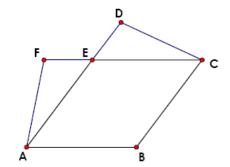
Prove: ABCD is a parallelogram.



Statements Reasons

4. Given: ABCE is a rhombus $\angle FAB \cong \angle DCB$

Prove: $\overline{FE} \cong \overline{DE}$



Statements Reasons

SUMMARY

QUADRILATERAL FAMILY

